## Problem of the week

## The gas laws

(a) Explain, in terms of molecular motion, the origin of pressure in a gas.

(b) The molar mass of helium is 4.0 g $mol^{-1}$.

- (i) Estimate the number of molecules in 3.0 g of helium.
- (ii) Determine the volume of 3.0 g of helium kept at pressure $1.0 \times 10^5$ Pa and temperature 320 K.
- (iii) Estimate the volume in (ii) that corresponds to one helium molecule.
- (iv) Estimate, using the answer to (iii) the average separation of helium molecules.

(c) The radius of a helium molecule is about 31 pm.

- (i) Calculate the actual volume occupied by the helium molecules.
- (ii) By reference to a specific assumption in the kinetic theory of ideal gases, suggest whether there is evidence that helium behaves as an ideal gas.

(d)

- (i) Calculate the density of helium in (b).
- (ii) Determine the average speed of helium molecules in (b).
- (iii) Hence or otherwise estimate the mass of a helium molecule.

(e) The pressure of the gas in (b) is increased to $3.0 \times 10^5$ Pa at constant volume. Determine the change in the internal energy of the gas.

## Answers

(a) The molecules collide with the container walls and their momentum is changed. Thus, the wall exerts a force on the molecules. Hence, by Newton's third law the molecules exert a force on the container walls and hence a pressure.

(b)

(i) The number of moles is $\frac{3.0}{4.0} = 0.75$ so the number of molecules is $0.75 \times 6.02 \times 10^{23} = 4.515 \times 10^{23} \approx 4.5 \times 10^{23}$ .

(ii) $PV = nRT \Rightarrow V = \frac{nRT}{P} = \frac{0.75 \times 8.31 \times 320}{1.0 \times 10^5} = 1.994 \times 10^{-2} \approx 2.0 \times 10^{-2}\ \text{m}^3$.

(iii) $\frac{1.99 \times 10^{-2}}{4.51 \times 10^{23}} = 4.416 \times 10^{-26} \approx 4.4 \times 10^{-26}\,\text{m}^3$.

(iv) $\sqrt[3]{4.416 \times 10^{-26}} \approx 3.5 \times 10^{-9}\ \text{m}$ .

(c)

(i) The volume of one molecule is $\frac{4\pi}{3}R^3 = \frac{4\pi}{3} \times (31 \times 10^{-12})^3 = 1.248 \times 10^{-31} \approx 1.25 \times 10^{-31}\,\text{m}^3$ and so the total volume of molecules is $4.515 \times 10^{23} \times 1.248 \times 10^{-31} = 5.672 \times 10^{-8} \approx 5.7 \times 10^{-8}\ \text{m}^3$ .

(ii) The relevant assumption is that the volume occupied by the molecules is negligible compared to the volume of the gas itself. This is the case here.

(d)

(i) $\rho = \frac{3.0 \times 10^{-3}}{1.994 \times 10^{-2}} = 0.15045 \approx 0.15\ \text{kg m}^{-3}$

(ii) $P = \frac{1}{3}\rho c^2 \Rightarrow c = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.0 \times 10^5}{0.15045}} = 1.412 \times 10^3 \approx 1.4 \times 10^3\ \text{m s}^{-1}$

(iii) $\frac{1}{2}mc^2 = \frac{3}{2}kT \Rightarrow m = \frac{3kT}{c^2} = \frac{3 \times 1.38 \times 10^{-23} \times 320}{(1.412 \times 10^3)^2} \approx 6.6 \times 10^{-27}\ \text{kg}$ ***OR***

$m = \frac{4.0 \times 10^{-3}}{6.02 \times 10^{23}} \approx 6.6 \times 10^{-27}\ \text{kg}$ .

(e) The new temperature is: $\frac{1.0 \times 10^5}{320} = \frac{3.0 \times 10^5}{T} \Rightarrow T = 960\ \text{K}$. The change in internal energy is then

$$\Delta U = \frac{3}{2}RnT_f - \frac{3}{2}RnT_i = \frac{3}{2}Rn\Delta T = \frac{3}{2} \times 8.31 \times 0.75 \times (960 - 320) = 6.0 \times 10^3\ \text{J}.$$